

## Mathematical description of the production of extruded products enriched with nut flour

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### Summary

The article considers the optimization problem of the production of extruded products enriched with nut flour. Based on the data obtained as a result of multifactorial experiment, the influence of all the factors involved in the process on the functional and technological parameters that determine the quality of the product was studied. The influence of each factor on bulk weight, expansion rate and mechanical energy density inputs of extrudates was determined using MathCad 15 program (Mathsoft, Cambridge, Massachusetts, USA). The experimental functions of the dependences on the above parameters of all these factors were selected and regression relationships reflecting the process were obtained. The methodology for calculating some of the process parameters was given, which allowed the results of experimental studies to be generalized to a production extruder. The results obtained from laboratory studies using the similarity theory and the scale-up method were transferred to industrial extruders for the production of extrudates with a porous structure. Research allows to design and produce extrudates with pre-defined functional properties and quality indicators having an appropriate structure, as well as to make preliminary calculations of energy inputs.

### Keywords

extrudate; process parameter; starch; nut flour; mathematical description; optimal parameter

Various supplements of plant and animal origin that are based on starch-containing raw materials are used to produce extrudates with various composition, as well as having different functional and technological properties [1]. The wide range of raw materials and the proper selection of processing conditions allow us to produce new products with high nutritional and biological value with pre-defined functional properties, by applying the innovative technology of thermoplastic extrusion [2–4].

Development of formulas of new extrusion products is based on the multi-factor experimental design methodology [2, 5–7]. Therefore, the use of raw materials and supplements of domestic origin for the production of new extruded products for national market becomes urgent. The production of any product is always associated with profitability [8], which relates to the process optimization

problem. In case of this study, issues of optimization of the production of extruded products enriched with nut flour are relevant and are the aim of scientific research along these lines. On the other hand, scientific research is one topic, while implementation of the results obtained is another one. Therefore, it is important to develop methods that will facilitate production of new products under industrial conditions. The results of optimizing the production process of extruded products enriched with nut flour can be used to develop a thermoplastic extrusion process management system to obtain a product with pre-defined functional and technological properties.

The topic of this research was the experimental and theoretical study of the process of production of promising products enriched with walnut crops based on starch-containing raw materials by thermoplastic extrusion.

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## MATERIALS AND METHODS

For the purpose of carrying out experimental studies, in accordance with the formulations, the authors of the article have selected materials as follows: cornflakes, maize starch according to ISO 11085:2015 [9], walnuts according to DDP-02 [10], peanuts according to ISO 6478:1990 [11] and table salt according to CODEX STAN 150-1985 [12].

Samples of extrudates for the studies were taken from an extruder K-30 (Kiiko, Dnepropetrovsk, Ukraine) composed of extrusion chamber, auger kit, forming die with various matrix diameters and the control panel. The extruder chamber was a hollow cylinder of 400 mm length and 19 mm inner diameter of the auger, with 6 longitudinal channels designed to transport the processed mass when using raw materials with a floury structure. On the outer surface of the cylinder, two heating elements were mounted. In general, the extruder has three zones, namely, the mixing, plasticizing and charging zones. From the top of the cylinder, a vertical single-screw proportioning feeder with a pyramid hopper is secured. Inside the chamber, a single-thread variable-pitch auger with an outer diameter of 19 mm is placed. During the studies, we used the auger kit with varying values of charging. At the end of the extruder chamber, a forming die with a matrix is connected by a threaded connection. We used matrices with various hole diameters. We varied the auger speed from  $150 \text{ min}^{-1}$  to  $230 \text{ min}^{-1}$ , and measured the rotary speed by means of a tachometer. The temperature in the cylinder was measured using a thermocouple.

In preliminary studies, the extrusion mixture contained 56.8 % cornflakes, 10 % maize starch, 12 % walnuts, 4.5 % peanuts, 0.7 % table salt and the moisture content of the mixture, with added water, was 16 %. This ratio of the components of the extrusion mixture was determined by a method of sensory analysis of taste, colour and rigidity. During the experiments, the moisture content of mixture varied between 16 % and 35 %.

The mixture components were hydrated before the extrusion process and it was settled at a temperature of  $+5^\circ\text{C}$  for 24 h. The mixture was extruded for the determined values of moisture content, temperature, auger speed, auger types and matrix size. A temperature of  $70^\circ\text{C}$  was maintained in the extruder's feed zone. After the extruder was operated in a stable mode, samples were taken, their volume weights and expansion rates were determined and, in parallel, the mechanical energy density expenditure was calcu-

lated. The volume weights of extrudates were determined as follows: vessels of a previously-known capacity of  $0.5 \times 10^{-3} \text{ dm}^3$  were filled with extrudates and then weighed on an analytical balance. The bulk weight ( $\rho$ ) was calculated by the following formula:

$$\rho = \frac{G}{V} \quad (1)$$

where  $G$  is weight of extrudates (in kilograms) and  $V$  is the volume occupied by extrudates (in cubic metres).

The sample weight of the bean before and after heat treatment was determined using SF-400C scales (Toms, Qilin, China) with a weighing accuracy of 0.01 g.

The expansion rate ( $Ex$ ) of extrudates was calculated as follows

$$Ex = \frac{D}{d} \quad (2)$$

where  $D$  is extrudate diameter (in millimetres) and  $d$  is the matrix die hole diameter (in millimetres).

Linear dimensions of the bean were determined using electronic digital caliper Vinca DCLA-0605, 150 mm. (Neiko Tools, Lu Chu Hsiang, Taiwan).

Mechanical energy density ( $E$ ) was calculated using the formula

$$E = M \cdot \frac{n}{Q} \quad (3)$$

where  $M$  is auger torque moment (in Newton metres),  $n$  is auger's number of rotations (in reciprocal minutes) and  $Q$  is extruder capacity (in kilograms per hour).

To further optimize the process of obtaining the base product, we used mathematical multi-factorial experiment design. The main factors for obtaining the base product were the moisture content of the extrudate mixture ( $W$ ), the temperature in the extruder's forming die ( $T$ ), the auger's number of rotations ( $n$ ), charging degree of extrudate ( $S$ ) and the matrix diameter ( $d$ ). To further optimize the process in the case of five-factorial experiments, it is necessary to conduct 25 experiments and vary each factor at five levels [13]. The range of factor variation was selected based on our previous research on model systems.

In order to optimize the process, mathematical methodology of multi-factor experimental design was used [14]. Main factors of the process were identified, their variation ranges were determined (Tab. 1) and characteristic functional (performance) indicators of the product selected, which were the optimization problem criteria. As such

characteristic values, specific weight of the product and its expansion rate were identified. The scheme of the experiment according to our early research is given in Tab. 2.

The relationships between functional indicators of extrudates and the process parameters were mathematically described using MathCad program (Mathsoft, Cambridge, Massachusetts, USA). We used the following functions for mathematical evaluation:

$$Y = ax + b \quad (4)$$

$$Y = a + \frac{b}{x} \quad (5)$$

$$Y = a^x + b \quad (6)$$

$$Y = a + \exp(bx) \quad (7)$$

$$Y = a + bx + cx^2 \quad (8)$$

$$Y = \frac{1}{(a + bx + cx^2)} \quad (9)$$

The value  $\tau$  of the function  $Y$  was determined according to the Eq. 10.

$$\tau = R \cdot \frac{\sqrt{N - K - 1}}{1 - R^2} \quad (10)$$

where

$$R = \sqrt{1 - (N - 1) \times \frac{\sum_{i=1}^N (Y_e - Y_p)^2}{(N - K - 1) \cdot \sum_{i=1}^N (Y_e - Y_a)^2}} \quad (11)$$

where  $R$  is a non-linear multi-way correlation coefficient,  $N$  is the number of levels,  $K$  is the number of functions influencing the process,  $Y_e$  is the experimental value of the function,  $Y_p$  is the value obtained through the function calculation, and  $Y_a$  is the average value of non-linear function.

The function  $Y$  is considered significant if  $\tau > 2$ . The generalized functions  $Y_g$  of the multifactorial dependences were calculated according to the formula [13]:

$$Y_g = \frac{\prod_1^N Y_i}{Y_{ag}^{n-1}} \quad (12)$$

where  $Y_g$  is a generalized function,  $Y_i$  is a partial function,  $\prod_1^N Y_i$  is partial function's derivative and  $Y_{ag}$  is the average value of all the generalized functions.

### Statistical analysis

To analyse the test parameters (the moisture content of starch paste, gelatinization point, starch paste transparency, starch paste embrittlement temperature and starch paste modulus of elas-

**Tab. 1.** Main factors of the process.

No.	Factor	Level $j$				
		1	2	3	4	5
1	$W$ [%]	15	20	25	30	35
2	$T$ [°C]	150	160	170	180	190
3	$n$ [min <sup>-1</sup> ]	150	170	190	210	230
4	$S$	1	2	3	4	5
5	$d$ [mm]	2	3	4	5	6

$W$  – moisture of the mixture,  $T$  – temperature in the extruder cylinder,  $n$  – auger's number of rotations,  $S$  – charging degree of extrudate,  $d$  – matrix hole diameter.

**Tab. 2.** The experiment performance grid.

No.	$W$ [%]	$T$ [°C]	$n$ [min <sup>-1</sup> ]	$S$	$d$ [mm]
1	15	150	150	1	2
2	15	160	190	3	4
3	15	170	170	2	3
4	15	180	230	5	6
5	15	190	210	4	5
6	20	150	170	4	4
7	20	160	230	1	3
8	20	170	210	3	6
9	20	180	150	2	5
10	20	190	190	5	2
11	25	150	170	2	6
12	25	160	190	5	5
13	25	170	230	4	2
14	25	180	210	1	4
15	25	190	150	3	3
16	30	150	210	5	3
17	30	160	150	4	6
18	30	170	190	1	5
19	30	180	170	3	2
20	30	190	230	2	4
21	35	150	230	3	5
22	35	160	210	2	4
23	35	170	150	5	2
24	35	180	190	4	3
25	35	190	170	1	6

$W$  – moisture of the mixture,  $T$  – temperature in the extruder cylinder,  $n$  – auger's number of rotations,  $S$  – charging degree of extrudate,  $d$  – matrix hole diameter.

ticity) of extrusion products, statistical analysis of the obtained data was carried out, and reliability of these data was evaluated by  $T$ -test using SPSS Statistics version 20.0 program (IBM, Armonk, New York, USA). To describe the ordered sample, we used the statistical functions of arithmetic mean of five experiments at each level and standard error of the mean. We chose a reliability value of  $p < 0.05$ .

## RESULTS AND DISCUSSION

Based on the experimental studies, we determined the relationship between the process parameters and the functional as well as physical characteristics of extrudates enriched with nut flour. The results of these studies are given in Tab. 3. The results obtained are consistent with the results obtained by other authors [2, 4, 15–17].

The obtained experimental results were described by means of various types of mathematical functions. Based on calculations, the best way to describe the relationship between the process parameters and the functional as well as physical characteristics of extrudates was the use of a second-order polynome:

$$Y = ax + bx + c \quad (13)$$

This was proven by a higher correlation coefficient than for other functions used, with the exception of the relationship between the expansion rate of extrudates and the number of rotations of the extruder's auger, which was better described by means of the function

$$Y = \frac{a}{x} + b \quad (14)$$

The values of the bulk weight, expansion rate and mechanical energy density of extrudates calculated by means of the selected functions are

given in Tab. 4. These results were close to those obtained in the other studies [4, 17, 18].

The comparison of experimental and computational results allowed us to conclude that these values were close to each other, indicating the correctness of selection of these functions [2–4, 16, 18, 19]. Based on the calculations performed, generalized relationships (functions) were obtained that describe changes in functional indicators and mechanical energy density based on process parameters. These generalized functions are presented in Tab. 5. Similar evaluations were previously obtained by other authors [20–22]. The difference in the ratios was apparently due to different types of extruders and raw materials used.

These dependences are not “one-size-fits-all” for describing the thermoplastic extrusion process, but they can be used to predict the values of functional and physical indicators of extrudates with a fairly high degree of accuracy in the selected ranges of process parameters [23]. They allow us to determine the impact of each factor on the above-mentioned indicators.

Based on previous studies [23, 24], using particular and generalized functions, we determined optimal conditions for the thermoplastic extrusion process of hazelnut flour-enriched extruded products:

**Tab. 3.** The relationship between the process parameters and the functional as well as physical characteristics of extrudates enriched with nut flour.

Function	Parameter	Level <i>j</i>					Average value
		1	2	3	4	5	
Bulk weight [10 <sup>-3</sup> kg·m <sup>-3</sup> ]							
ρ <sub>1j</sub>	<i>W</i>	0.510	0.103	0.211	0.308	0.358	0.226
ρ <sub>2j</sub>	<i>T</i>	0.249	0.277	0.236	0.204	0.215	0.226
ρ <sub>3j</sub>	<i>n</i>	0.216	0.222	0.231	0.252	0.210	0.226
ρ <sub>4j</sub>	<i>S</i>	0.275	0.241	0.200	0.192	0.223	0.226
ρ <sub>5j</sub>	<i>d</i>	0.215	0.203	0.237	0.234	0.242	0.226
Expansion rate							
<i>Ex</i> <sub>1j</sub>	<i>W</i>	1.930	2.640	2.190	2.020	1.840	2.12
<i>Ex</i> <sub>2j</sub>	<i>T</i>	2.180	2.200	2.060	2.110	2.070	2.12
<i>Ex</i> <sub>3j</sub>	<i>n</i>	1.940	2.210	2.140	2.040	2.280	2.12
<i>Ex</i> <sub>4j</sub>	<i>S</i>	1.950	1.960	2.140	2.220	2.340	2.12
<i>Ex</i> <sub>5j</sub>	<i>d</i>	2.270	2.590	2.140	1.960	1.650	2.12
Mechanical energy density [10 <sup>-3</sup> J·kg <sup>-1</sup> ]							
<i>E</i> <sub>1j</sub>	<i>W</i>	9.940	7.728	8.906	5.958	3.994	7.305
<i>E</i> <sub>2j</sub>	<i>T</i>	6.602	8.908	7.700	6.596	6.720	7.305
<i>E</i> <sub>3j</sub>	<i>n</i>	5.996	6.602	7.788	6.404	9.736	7.305
<i>E</i> <sub>4j</sub>	<i>S</i>	8.108	5.732	5.772	9.934	6.980	7.305
<i>E</i> <sub>5j</sub>	<i>d</i>	8.284	9.194	5.388	8.082	5.578	7.305

*W* – moisture of the mixture, *T* – temperature in the extruder cylinder, *n* – auger's number of rotations, *S* – charging degree of extrudate, *d* – matrix hole diameter.

**Tab. 4.** Calculated values of bulk weight, expansion rate and mechanical energy density of extrudates.

Function	Calculated values of a function at level <i>j</i>					Average value
	1	2	3	4	5	
<b>Bulk weight</b> [10 <sup>-3</sup> kg·m <sup>-3</sup> ]						
$\rho_1 = 0.22123 - 0.01407W + (0.528 \times 10^{-3})W^2$	0.129	0.151	0.200	0.275	0.376	0.226
$\rho_2 = 0.92108 - 0.00732T + (0.019 \times 10^{-3})T^2$	0.248	0.233	0.222	0.215	0.212	0.226
$\rho_3 = -0.32330 + 0.00582n - (0.015 \times 10^{-3})n^2$	0.210	0.230	0.238	0.234	0.218	0.226
$\rho_4 = 0.35440 - 0.08569S + 0.01171S^2$	0.280	0.230	0.203	0.199	0.219	0.226
$\rho_5 = 0.19392 + 0.00762d + 0.0001d^2$	0.210	0.218	0.226	0.234	0.243	0.226
<b>Expansion rate</b>						
$Ex_1 = 0.02566 + 0.20114W - (0.434 \times 10^{-3})W^2$	2.066	2.311	2.340	2.151	1.746	2.122
$Ex_2 = 3.68840 - 0.01532T + (0.036 \times 10^{-3})T^2$	2.194	2.151	2.116	2.087	2.066	2.122
$Ex_3 = 2.61048 - 90.57343 \cdot n^{-1}$	2.007	2.078	2.134	2.180	2.217	2.122
$Ex_4 = 1.85840 + 0.06351S + 0.00671S^2$	1.929	2.012	2.109	2.220	2.344	2.122
$Ex_5 = 1.86920 + 0.38611d - 0.07171d^2$	2.354	2.382	2.266	2.007	1.604	2.122
<b>Mechanical energy density</b> [10 <sup>-3</sup> J·kg <sup>-1</sup> ]						
$E_1 = 8.17263 + 0.24533W - 0.01037W^2$	9.519	8.931	7.824	6.198	4.054	7.305
$E_2 = -76.4956 + 1.01381T - (3.04 \times 10^{-3})T^2$	7.112	7.817	7.914	7.402	65.281	7.305
$E_3 = 18.55419 - 0.15915n - (0.51 \times 10^{-3})n^2$	6.260	6.371	6.893	7.828	9.173	7.305
$E_4 = 8.20440 - 1.07655S + 0.21186S^2$	7.340	6.899	6.881	7.288	8.118	7.305
$E_5 = 9.58680 - 0.46497d - 0.02343d^2$	8.563	7.981	7.352	6.676	5.954	7.305

$W$  – moisture of the mixture,  $T$  – temperature in the extruder cylinder,  $n$  – auger's number of rotations,  $S$  – charging degree of extrudate,  $d$  – matrix hole diameter.

**Tab. 5.** Generalized functions of the process parameters.

Generalized function	Equation	Eq.
Bulk weight [ $10^{-3} \text{ kg} \cdot \text{m}^{-3}$ ]	$\rho = \frac{[0.22123 - 0.001407W + (0.528 \times 10^{-3})W^2] \times [0.92108 - 0.00732T + (0.019 \times 10^{-3})T^2]}{0.2263 \times [(0.3544 + 0.0856S + 0.0117S^2) \times (0.1939 + 0.00762d + 0.0001d^2)]^{-1}}$	15
Expansion rate	$Ex = \frac{[0.0256 + 0.2011W - (4.34 \times 10^{-3})W^2] \times [3.6884 - 0.0153T + (0.036 \times 10^{-3})T^2]}{2.123 \times [(1.8584 + 0.0635S + 0.00671S^2) \times (1.8692 + 0.03861d - 0.0717d^2)]^{-1}}$	16
Mechanical energy density [ $10^{-3} \text{ J} \cdot \text{kg}^{-1}$ ]	$E = \frac{(8.1726 + 0.2425W - 0.0104W^2) \times [18.5542 - 0.15915n + (0.5 \times 10^{-3})n^2]}{7.305}$	17

$W$  – moisture of the mixture,  $T$  – temperature in the extruder cylinder,  $n$  – auger's number of rotations,  $S$  – charging degree of extrudate,  $d$  – matrix hole diameter.

$$\rho(W; T; S; d) = \min. \quad (18)$$

$$Ex(W; T; S; d) = \max. \quad (19)$$

By differentiating the experimental partial functions, taking into account the generalized, organoleptic and visual evaluations (the best organoleptic value was observed at bulk weight  $\rho = 113 \text{ kg} \cdot \text{m}^{-3}$  and at the expansion ratio of  $Ex = 3$ ), we determined optimal values of the extrusion process parameters. As data in Tab. 6 indicate, the optimal values of the parameters calculated for both functional indicators were close to each other. This indicated that the optimization problem could be solved by one of these functional indicators. This is in good agreement with other literature data [4, 5, 17, 21, 24, 25].

To calculate the mechanical energy density inputs, we set the derivative of the corresponding partial function to zero

$$\frac{\partial E(n)}{\partial n} = (1.130 \times 10^{-3})n - 0.159 = 0 \quad (20)$$

where  $n = 155 \text{ min}^{-1}$ .

Then, we determined the mechanical energy density  $E$  input taking into account all the process parameters

$$E = E(W) \times E(T) \times E(n) \times E(S) \times \frac{E(d)}{E_a^4} \quad (21)$$

$$E = (7.17 \times 10^3) \text{ J} \cdot \text{kg}^{-1}$$

where  $E_a$  is the average value of mechanical energy density (in joules per kilogram).

The results obtained in laboratory studies



**Tab. 6.** Optimal values of the extrusion process parameters.

Process parameters	Functional properties of the base product	
	$\rho = 113 \text{ kg}\cdot\text{m}^{-3}$	$Ex = 3$
$W$ [%]	20	20
$T$ [°C]	190	188
$n$ [min <sup>-1</sup> ]	170	170
$S$	4 : 1	5 : 1
$d$ [mm]	3	3

$W$  – moisture of the mixture,  $T$  – temperature in the extruder cylinder,  $n$  – auger's number of rotations,  $S$  – charging degree of extrudate,  $d$  – matrix hole diameter,  $\rho$  – bulk weight,  $Ex$  – expansion rate.

were generalized using the theory of similarity and the scale-up method for the production of these products on industrial extruders. It is known that there are similarities between the technological processes if they have geometrical and time-related similarities, similarities between the fields of physical quantities, as well as similarities between the initial and boundary conditions [7, 14, 17, 21]. Process parameters such as the moisture content of raw materials and their treatment temperature can be directly transferred to the production conditions, so the moisture content of 20 % of raw materials and the temperature of 190 °C in the laboratory extruder cylinder are the operating parameters of the industrial extruder. As for the matrix diameter, the compression ratio of raw materials from the auger and the number of its rotations, the influence of these factors on the extrusion process is expressed by the process duration and the stresses induced by mechanical impact on raw materials. The raw materials processing time ( $t$ ) in the extruder is determined according to the equation

$$t = \frac{V}{Q_v} \quad (22)$$

where  $V$  is the volume of the processed mass in the extruder (in cubic metres),  $Q_v$  is the volumetric production of the extruder (in cubic metres per hour).

It should be noted that similarity between the values of processing time in the laboratory and production extruders requires serious changes in the equipment drive system, but there is an inverse correlation between the processing time and mechanical stress that affect the functional properties of extrudates. Thus, insufficient time required for raw materials processing can be filled out by more intense mechanical treatment. Mechanical impact on processed raw materials can be evaluated (with high accuracy) by the shear-stress rate ( $v_{\sigma}$ ) of the

fluid (molten) mass moving in the extruder. For the circular hole of the extruder's matrix, it is calculated according to the equation and expressed in metres per second

$$v_{\sigma} = 32 \frac{Q_v}{\pi d^3 K} \quad (23)$$

where  $d$  is matrix diameter (in metres) and  $K$  is the number of matrix holes.

In view of the above, the following equation is valid:

$$\frac{t_1}{t_2} = \frac{v_{\sigma 2}}{v_{\sigma 1}} \quad (24)$$

where  $t_1$  and  $t_2$  is raw materials processing time in the laboratory and industrial extruders, respectively (in seconds),  $v_{\sigma 1}$  and  $v_{\sigma 2}$  is the shear-stress rate of the fluid mass in the laboratory and industrial extruders, respectively (in metres per second). On the other hand, the raw materials processing time in the extruder is equal to the total lag time of masses passing through its cylinder, moulding die and matrix (maximum shear deformation zone), that is why we used in calculations the lag time  $t$  of the fluid mass in the matrix.

Knowing the shear-stress rate of the processed mass, its lag time in the matrix hole channel of the laboratory extruder, as well as the technical data of the industrial extruder, the required diameter of the matrix hole of the industrial extruder can be determined. In particular

$$t_m = \frac{L_m}{v_m} \quad (25)$$

where  $t_m$  is the residence time in the fluid mass matrix (in seconds),  $L_m$  is matrix length (in metres),  $v_m$  is the the speed of the extrusion mass (in metres per second).

The fluid mass rate in the matrix was determined based on the principle of conservation of mass

$$Q_c = Q_d = Q_m \quad (26)$$

$$\rho v_c F_c = \rho v_d F_d = \rho v_m F_m \quad (27)$$

where  $Q_c$ ,  $Q_d$ ,  $Q_m$  are the mass productions in the extruder's cylinder, molding die and matrix, respectively (in kilograms per second),  $F_c$ ,  $F_d$ ,  $F_m$  are the areas of the extruder's cylinder, moulding die and matrix, respectively (in square metres) and  $v_c$ ,  $v_d$ ,  $v_m$  are the speed of the extrusion mass in the extruder's cylinder, moulding die and matrix respectively (in metres per second).

Then

$$\frac{v_{\sigma 2}}{v_{\sigma 1}} = \frac{\sigma_2}{\sigma_1} = \frac{E_2}{E_1} \quad (28)$$

where  $\sigma_1$  and  $\sigma_2$  are mechanical stresses in the laboratory and industrial extruders, respectively (in Pascals),  $E_1$  and  $E_2$  are mechanical energy densities inputs in the laboratory and industrial extruders, respectively (in joules per kilogram).  $E_1$  was determined based on the optimization condition and for the industrial extruder,  $E_2$  was determined according to the formula

$$E_2 = \frac{N_2}{Q_2} \quad (29)$$

where  $N_2$  is industrial extruder capacity (in joules per second) and  $Q_2$  is the mass production of industrial extruders (in kilograms per second).

Finally, from Eq. 30, we found the required diameter of the selected extruder's matrix hole for the production of extrudates with optimal functional indicators.

$$d = \sqrt[3]{32 \frac{Q_{v2}}{\pi v_{\sigma 2} K_2}} \quad (30)$$

## CONCLUSION

Based on the conducted studies, we could identify the process parameters that can be used to regulate the functional properties of extruded products enriched with nut flour. The optimal parameters of the process of thermoplastic extrusion for the production of extrudates enriched with nut flour were determined. The generalized mathematical dependences for the process to determine the functional characteristics of extrudates ( $\rho = 113 \text{ kg}\cdot\text{m}^{-3}$  and  $Ex = 3$ ). The methodology for calculating certain parameters, with a view to choose the industrial extruder, was developed.

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